

Correspondence of consistent and inconsistent spin-3/2 couplings via the equivalence theorem

V. Pascalutsa

Department of Physics, Flinders University, Bedford Park, SA 5042, Australia

(December 12, 2000)

Abstract

The Rarita-Schwinger theory of free massive spin-3/2 field obeys the physical degrees of freedom counting, and one can distinguish “consistent” couplings which maintain this property and “inconsistent” couplings which destroy it. We show how one can obtain consistent couplings from inconsistent ones by a redefinition of the spin-3/2 field. The field redefinition gives rise to additional “contact terms” which, via the equivalence theorem, can be associated with the contribution of the extra lower-spin degrees of freedom involved by the inconsistent coupling.

Relativistic description of hadrons and their dynamics often faces with the problem of higher-spin (HS) fields. A covariant HS field has more components than is necessary to describe the spin degrees of freedom (DOF) of the physical particle, and therefore in formulating the action certain symmetries must be imposed to reduce the number of DOF to the physical value. So any free massless HS action must be gauge symmetric [1], leaving only 2 spin DOF as is appropriate for a massless particle with spin. The mass term should (partially) break the gauge symmetries such that the number of DOF becomes $2s + 1$.

Consequently, not every form of the interaction will be consistent with the free theory construction. The “inconsistent” interaction will violate the DOF counting by involving the redundant lower-spin sector of the HS field. This gives rise to the “lower-spin background” contributions in observables, and, in some cases, to the presence of negative norm states [2] and acausal modes [3]. Many general forms of the interaction are ruled out due to exhibiting some of these pathologies (see *e.g.* [4–6]).

In recent work, focusing on the spin-3/2 field of the $\Delta(1232)$ isobar, we have argued [7,8] that the interactions with the same type of gauge-symmetry as the kinetic term of the spin-3/2 field do not change the DOF content of the free theory, hence are “consistent”; constructed *gauge-symmetric* $\pi N\Delta$ and $\gamma N\Delta$ couplings; and shown that they give rise to purely spin-3/2 contributions. On the other hand, during the past three decades, a vast number of studies has been done, see *e.g.* [9], using the conventional Δ couplings which are inconsistent from the viewpoint of the DOF-counting. Contributions due to the spin-1/2 sector of the Δ field, so-called “spin-1/2 backgrounds”, play in these studies an important, sometimes crucial role while their physical significance is unclear.

In this Letter we find the correspondence between the “inconsistent” and the “consistent” (gauge-invariant) couplings. The two can be related by a redefinition of the spin-3/2 field. The redefinition gives also rise to some higher-order, in the coupling constant, interactions (“contact terms”). As the latter are taken into account, the two theories — the inconsistent and the corresponding consistent one — lead to the same S -matrix elements (observables) in accordance with the equivalence theorem [10,11].

Thus, on one hand, we can establish the connection between the studies based on gauge and conventional Δ couplings. On the other hand, the spin-1/2 sector involved by the inconsistent coupling can be represented via certain contact interactions, and depending on the framework one might find a better physical interpretation of the “spin-1/2 backgrounds”. For instance, it is easy to understand how at the *tree level* the spin-1/2 sector of the Δ exchange can be reabsorbed into some meson and spin-1/2 baryon exchange contributions [12], or into some of the “low-energy constants” of ChPT [13,14]. Present equivalence theorem extends to any linear coupling of the spin-3/2 field and to any number of loops.

We begin with the Lagrangian of the free massive spin-3/2 Rarita-Schwinger (RS) field¹

$$\mathcal{L}_{\text{RS}} = \bar{\psi}_\mu(x) \Lambda^{\mu\nu}(i\partial) \psi_\nu(x), \quad \Lambda^{\mu\nu}(i\partial) \equiv \gamma^{\mu\nu\alpha} i\partial_\alpha - m \gamma^{\mu\nu}. \quad (1)$$

Corresponding field equations are

$$\Lambda^{\mu\nu}(i\partial) \psi_\nu = 0 = \gamma_\mu \Lambda^{\mu\nu}(i\partial) \psi_\nu = \partial_\mu \Lambda^{\mu\nu}(i\partial) \psi_\nu. \quad (2)$$

or, equivalently, $(i\gamma \cdot \partial - m)\psi_\mu = 0 = \gamma \cdot \psi = \partial \cdot \psi$. The kinetic term is, up to a total derivative, invariant under the gauge transformation:

$$\psi_\mu(x) \rightarrow \psi_\mu(x) + \partial_\mu \epsilon(x), \quad (3)$$

where $\epsilon(x)$ is a spinor.

The covariant propagator of the RS field in the momentum space takes the well-known form:

$$S^{\mu\nu}(p) = \frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} \left[-\eta^{\mu\nu} + \frac{1}{3}\gamma^\mu \gamma^\nu + \frac{1}{3m}(\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m^2}p^\mu p^\nu \right], \quad (4)$$

and satisfies

$$S^{\mu\alpha}(p) \Lambda^{\beta\nu}(p) \eta_{\alpha\beta} = \Lambda^{\mu\alpha}(p) S^{\beta\nu}(p) \eta_{\alpha\beta} = \eta^{\mu\nu}. \quad (5)$$

¹Our conventions: metric tensor $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$; γ -matrices γ^μ , $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$; fully antisymmetrized products of γ -matrices $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu] = \gamma^\mu\gamma^\nu - \eta^{\mu\nu}$, $\gamma^{\mu\nu\alpha} = \frac{1}{2}\{\gamma^{\mu\nu}, \gamma^\alpha\} = i\varepsilon^{\mu\nu\alpha\beta}\gamma_\beta\gamma_5$, $\gamma^{\mu\nu\alpha\beta} = \frac{1}{2}[\gamma^{\mu\nu\alpha}, \gamma^\beta] = i\varepsilon^{\mu\nu\alpha\beta}\gamma_5$; spinor indices are omitted. Other forms of the spin-3/2 Lagrangian, obtained from (1) by redefinition: $\psi^\mu \rightarrow (\eta^{\mu\nu} - \frac{1}{2}[1 + A]\gamma^\mu\gamma^\nu)\psi_\nu$, involving a parameter A , are often used in the literature: *e.g.* [2,15,16]; for $A \neq -\frac{1}{2}$ this redefinition leads to equivalent Lagrangians, and any of them could be adopted without affecting our results.

In considering the interactions, we will focus on the *linear coupling* of the spin-3/2 field, *i.e.*,

$$\mathcal{L}_{\text{int}} = g \bar{\psi}_\mu j^\mu + g \bar{j}^\mu \psi_\mu, \quad (6)$$

where j can depend on fields other than ψ ; g is a coupling constant. Consistency with DOF-counting would impose a condition on j^μ . For example, if the coupling is to be symmetric under the gauge transformation (3), then j must be divergenceless: $\partial \cdot j = 0$. Suppose, however, that our j does not obey any such condition and the coupling is *inconsistent*.

We can make a field redefinition:

$$\psi_\mu(x) \rightarrow \psi_\mu(x) + g \xi_\mu(x), \quad (7)$$

which gives rise to a new linear coupling $\mathcal{L}'_{\text{int}}$ plus a quadratic (in the coupling constant) interaction \mathcal{L}_C :

$$\begin{aligned} \mathcal{L}_{\text{RS}} + \mathcal{L}_{\text{int}} &\rightarrow \mathcal{L}_{\text{RS}} + \mathcal{L}'_{\text{int}} + \mathcal{L}_C, \\ \mathcal{L}'_{\text{int}} &= g \bar{\psi} \cdot (j + \Lambda \cdot \xi) + \text{H.c.} \equiv g \bar{\psi} \cdot j' + \text{H.c.}, \\ \mathcal{L}_C &= g^2 [\bar{\xi} \cdot \Lambda \cdot \xi + \bar{\xi} \cdot j + \bar{j} \cdot \xi], \end{aligned} \quad (8)$$

and find that field ξ_μ can always be chosen such that the new linear coupling is *consistent*. Namely, we take

$$\xi_\mu = (m \gamma^{\mu\nu})^{-1} j^\nu = -\frac{1}{m} \mathcal{O}_{\mu\nu}^{(-1/3)} j^\nu \quad (9)$$

where $\mathcal{O}_{\mu\nu}^{(x)} \equiv \eta_{\mu\nu} + x \gamma_\mu \gamma_\nu$. Then

$$j'^\mu = \gamma^{\mu\nu\alpha} i \partial_\alpha \xi_\nu \quad (10)$$

and consistency condition $\partial \cdot j' = 0$ is explicitly obeyed.

Note that in this case ξ and hence \mathcal{L}_C are independent of ψ_μ . Therefore, we state that (i) an inconsistent linear coupling of a massive spin-3/2 can in general be transformed, by a redefinition of the spin-3/2 field, into a consistent coupling plus an additional quadratic coupling that does not involve the spin-3/2 field. Furthermore, if both the Lagrangian and the field transformation satisfy the conditions of the equivalence theorem [11], then (ii) the description in terms of \mathcal{L}_{int} or $\mathcal{L}'_{\text{int}} + \mathcal{L}_C$ are equivalent at the level of S -matrix.

Moving the second-order coupling \mathcal{L}_C to the other side of the equation, we obtain a corollary of statements (i) and (ii): given any inconsistent linear coupling we can find the supplementary second-order interaction which will provide us with the description of observables identical to the one with the consistent coupling. The contact interaction \mathcal{L}_C and the spin-1/2 sector raised by the inconsistent coupling can be put in a direct correspondence.

To demonstrate these statements we focus on the spin-3/2 coupling to a spin-0 and a spin-1/2 field. Such couplings are frequently used in describing the coupling of the decuplet baryons to the pion and nucleon. In particular, the conventional $\pi N \Delta$ coupling reads [15]:

$$\mathcal{L}_{\pi N \Delta} = g \bar{\psi}_\mu^i (\eta^{\mu\nu} + z \gamma^\mu \gamma^\nu) T_{ik}^a \Psi^k \partial_\nu \phi^a + \text{H.c.}, \quad (11)$$

where g is a dimensionfull coupling constant, and z is an “off-shell parameter”. Herein we have retained the isospin since it plays some role in what follows. The pseudo scalar fields ϕ^a , correspond to the pion $\pi^a = (\pi^+, \pi^-, \pi^0)$; spinors Ψ^k correspond to the nucleon $N^k = (p, n)$; the RS fields ψ_μ^i represent the $\Delta^i = (\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$; T^a denotes the isospin 1/2 to 3/2 transition matrices satisfying $T^{\dagger a} T^b = \frac{2}{3} \delta^{ab} - \frac{1}{3} i \varepsilon^{abc} \tau^c$; τ^c are the isospin Pauli matrices.

This is an “inconsistent” spin-3/2 coupling. It violates the DOF counting of the massive RS theory for any value of the off-shell parameter z . Only for the case of neutral particles (no isospin) and $z = -1$ the DOF-counting is correct [15].²

To find a corresponding consistent coupling we make the field transformation (7) with

$$\xi_\mu = -\frac{1}{m} \mathcal{O}_{\mu\varrho}^{(-1/3)} \mathcal{O}^{(z)\varrho\nu} T^a \Psi \partial_\nu \phi^a \quad (12)$$

and thus obtain a gauge-invariant linear coupling (exactly the one found earlier in [7]):

$$\mathcal{L}'_{\pi N \Delta} = -\frac{ig}{2m} \overline{G}_{\mu\nu} \gamma^{\mu\nu\lambda} T^a \Psi \partial_\lambda \phi^a + \text{H.c.}, \quad (13)$$

where $G_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$, plus the second-order term:

$$\mathcal{L}_{\pi\pi NN} = -\left(\frac{g}{m}\right)^2 \overline{\Psi} \mathcal{O}_{\varrho\mu}^{(x)} (\gamma^{\mu\nu\alpha} i \partial_\alpha + m \gamma^{\mu\nu}) \mathcal{O}_{\nu\sigma}^{(x)} T^{\dagger b} T^c \Psi (\partial^\varrho \phi^{\dagger b}) (\partial^\sigma \phi^c). \quad (14)$$

with $x = -\frac{1}{3}(1+z)$. Thus, $\mathcal{L}_{\text{RS}} + \mathcal{L}_{\pi N \Delta} \rightarrow \mathcal{L}_{\text{RS}} + \mathcal{L}'_{\pi N \Delta} + \mathcal{L}_{\pi\pi NN}$.

To check that the field transformation leaves the S -matrix invariant let us first consider some simplest matrix elements involving the two vertices:

$$\begin{aligned} \Gamma^{\mu a}(k) &\equiv \Gamma^\mu(k) T^a, \quad \Gamma^\mu(k) = g (\eta^{\mu\nu} + z \gamma^\mu \gamma^\nu) k_\nu \quad (\text{inconsistent}) \\ \tilde{\Gamma}^{\mu a}(k, p) &\equiv \tilde{\Gamma}^\mu(k, p) T^a, \quad \tilde{\Gamma}^\mu(k, p) = -(g/m) \gamma^{\mu\nu\alpha} k_\nu p_\alpha \quad (\text{consistent}). \end{aligned}$$

The Δ production amplitude is apparently the same for both vertices

$$\overline{u}(p') \Gamma^{\mu a}(p' - p) u_\mu(p) = \overline{u}(p') \tilde{\Gamma}^{\mu a}(p' - p, p) u_\mu(p) = g \overline{u}(p') (p' - p)^\mu u_\mu(p) T^a, \quad (15)$$

where $u(p)$ is the nucleon spinor, $u_\mu(p)$ is the free RS vector-spinor satisfying $(\not{p} - m)u_\mu = 0 = p \cdot u = \gamma \cdot u$, for $p^2 = m^2$.

However for the Δ -exchange amplitudes in pion-nucleon scattering, Fig. 1, the two vertices yield quite different results. The inconsistent coupling involves the spin-1/2 sector of the RS propagator, and therefore the exchange amplitude,

²In that case one finds merely the Johnson-Sudarshan and Velo-Zwanziger problems [16]. However, these latter problems will not be addressed here since we use the “naive” Feynman rules (as is usually done!), hence ignoring the field-dependent determinants in the path-integral measure which arise in the constrained quantization procedure [7] and which contain the problem. In this usual “naive” interpretation, coupling (11) for neutral particles and $z = -1$ is “consistent”.

$$\Gamma^\mu(k') S_{\mu\nu}(p+k) \Gamma^\nu(k) \sim \frac{g^2}{m - (p+k) \cdot \gamma} P_{\mu\nu}^{3/2}(p+k) k'^\mu k^\nu + \text{“spin-1/2 background”}, \quad (16)$$

contains the controversial spin-1/2 background contributions, in addition to the spin-3/2 propagation represented by the spin-3/2 projection operator:

$$P_{\mu\nu}^{3/2}(p) = \eta_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3p^2} (\not{p} \gamma_\mu p_\nu + p_\mu \gamma_\nu \not{p}). \quad (17)$$

In contrast, the consistent coupling, because of the property $p \cdot \tilde{\Gamma}(k, p) = 0$, gives rise to only the spin-3/2 propagation [7,8], namely

$$\tilde{\Gamma}^\mu(k', p_s) S_{\mu\nu}(p_s) \tilde{\Gamma}^\nu(k, p_s) = \frac{g^2}{m - \not{p}_s} \frac{p_s^2}{m^2} P_{\mu\nu}^{3/2}(p_s) k'^\mu k^\nu. \quad (18)$$

where $p_s = p + k = p' + k'$.

Because the decomposition into the pure spin-3/2 and spin-1/2 sector is nonlocal (the projection operators are singular at $p^2 = 0$), it is not at all obvious that the difference between the amplitudes (16) and (18) can be compensated by the local contact term $\mathcal{L}_{\pi\pi NN}$. However it indeed happens, as can most easily be seen for the case $z = -1$, when

$$\tilde{\Gamma}^\mu(k, p) - \Gamma^\mu(k) = -(g/m) \Lambda^{\mu\nu}(p) k_\nu. \quad (19)$$

Using this identity and Eq. (5) we find for the s -channel exchange the difference between the amplitudes (16) and (18) is

$$\begin{aligned} M_C^{(s)} &= [\tilde{\Gamma}(k', p_s) \cdot S(p_s) \cdot \tilde{\Gamma}(k, p_s) - \Gamma(k') \cdot S_{\mu\nu}(p_s) \cdot \Gamma(k)] T^{\dagger a} T^b \\ &= \left[-(g/m) (k' \cdot \Gamma(k) + \Gamma(k') \cdot k) + (g/m)^2 k' \cdot \Lambda(p_s) \cdot k \right] T^{\dagger a} T^b. \end{aligned} \quad (20)$$

A similar contribution comes from the u -channel graph such that the total difference sums up into

$$M_C^{(s)} + M_C^{(u)} = -(g/m)^2 (T^{\dagger a} T^b - T^{\dagger b} T^a) [\tfrac{1}{2} \gamma^{\mu\nu\alpha} (p + p')_\alpha + m \gamma^{\mu\nu}] k'_\mu k_\nu, \quad (21)$$

which is exactly canceled by the contact interaction $\mathcal{L}_{\pi\pi NN}$.³

Clearly, Green's functions which do not represent observable quantities need not be the same. For instance, the one-loop Δ self-energy will be different for the two couplings. However, at the level of the S -matrix the equivalence is restored. That is, the amplitude containing the self-energy with the consistent coupling (l.h.s. in Fig. 2) is identical to the

³Notably, for the case of *neutral pions*, obtained by neglecting the isospin complications and taking the real scalar field in Eq. (11), the contact term $\mathcal{L}_{\pi\pi NN}$ vanishes. As we have already remarked, for this case, coupling (11) with $z = -1$ is consistent with the DOF-counting [15], does not involve the spin-3/2 components, and hence needs no additional terms to match the corresponding gauge-invariant coupling.

one-loop amplitude with the inconsistent coupling plus the contact term (r.h.s. in Fig. 2). It is easy to convince oneself that this equivalence will persist to any number of loops.

In the analysis of the πN scattering we have recently noted numerically [12] that, once the ρ and σ meson exchange are included, the gauge-invariant Eq. (13) and conventional Eq. (11) couplings, at the tree level, give the same prediction for the πN threshold parameters provided some coupling constants are readjusted. This observation can now be understood as the very low-energy meson exchanges may take the role of the contact term $\mathcal{L}_{\pi\pi NN}$ needed to provide the equivalence between the couplings.

Another relevant statement with respect to the $\pi N\Delta$ coupling has been recently made by Tang and Ellis [13] from the point of view of Chiral Perturbation Theory (ChPT). They have shown that the contribution of the “off-shell parameter” to the Δ -exchange amplitude can be absorbed into contact terms which are already present in the ChPT Lagrangian. Therefore, they argue, this parameter is redundant in ChPT. By using the field redefinition we can confirm their result and extend it to any linear spin-3/2 couplings, *e.g.* the $\gamma N\Delta$ couplings.

It is further possible to argue that within the ChPT framework any linear spin-3/2 coupling is acceptable: in ChPT the additional \mathcal{L}_C type of terms, which provide the equivalence of “inconsistent” and “consistent” couplings, are to be included anyway with arbitrary coefficients and in both situations. (Thus, the effective Lagrangian with an inconsistent coupling and a \mathcal{L}_C term with arbitrary coefficient c_1 , is equivalent to the Lagrangian with a consistent coupling and \mathcal{L}_C term with a different but yet arbitrary coefficient $c_2 = c_2(c_1, g/m)$. The spin-1/2 background will lead only to a change in the values of some coupling constants.)

Nevertheless, let us emphasize the use of the gauge-invariant couplings makes the calculations much easier and more transparent. In particular, the spin-1/2 sector can be entirely dropped from the RS propagator, while analyzing the spin-3/2 self-energy, one does not need to consider the ten scalar functions of the most general tensor structure [17], but only two of them [18,12], just as in the spin-1/2 case.

Besides the technical advantages, gauge-invariant couplings involve the physical highest-spin contributions only, and hence they are preferable in the analysis of separate contributions and effects due to spin-3/2 particles versus the rest. This is important when the properties of separate HS resonances are being extracted in a model-dependent way from experimental data. Here we in particular keep in mind the current programs at CEBAF and MAMI aimed to measure various electromagnetic properties of the $\Delta(1232)$ isobar.

Much of the said about the linear couplings is applicable to the *quadratic*, etc., couplings as well. Given any inconsistent coupling of a massive RS field ψ_μ , we can obtain an on-shell equivalent consistent coupling by the replacement:

$$\psi_\mu \rightarrow \frac{i}{m} \mathcal{O}_{\mu\lambda}^{(-1/3)} \gamma^{\lambda\alpha\beta} \partial_\alpha \psi_\beta. \quad (22)$$

It is then possible to work out the exact field transformation relating the couplings and the supplementary higher-order terms providing their equivalence at the S -matrix level. Working these out for any specific example is beyond the scope of this letter. Without going into details it is already clear that once the coupling involves several RS fields, the transformation must be nonlinear in the RS field and hence the number of supplementary terms is infinite.

In conclusion, we have shown that for any linear coupling of a massive spin-3/2 field all the possibly arising spin-1/2 contributions can be represented by a single interaction term independent of the spin-3/2 field itself. This is proven by showing that any “inconsistent” coupling involving the spin-1/2 components can be transformed by a field redefinition into a “consistent” gauge-invariant coupling that decouples the spin-1/2 sector. The additional interaction term arises in the course of field transformation and is needed to provide the equivalence of the two couplings at the level of S -matrix elements. We emphasize that consistent interactions have conceptual and technical advantages, making the calculations simpler and with more transparent interpretation.

ACKNOWLEDGMENTS

I am very thankful to Jambul Gegelia for many interesting discussions, and to Iraj Afnan, Stanley Deser, Thomas Hemmert, Justus Koch, Andrew Waldron for valuable remarks. The work is supported by the Australian Research Council (ARC).

REFERENCES

- [1] M. Fierz and W. Pauli, “On Relativistic Wave Equations For Particles Of Arbitrary Spin In An Electromagnetic Field,” *Proc. Roy. Soc. Lond.* **A173** (1939) 211; W. Rarita and J. Schwinger, “On A Theory Of Particles With Half Integral Spin,” *Phys. Rev.* **60** (1941) 61; L. P. Singh and C. R. Hagen, “Lagrangian Formulation For Arbitrary Spin,” *Phys. Rev.* **D9** (1974) 898, 910; B. de Wit and D. Z. Freedman, “Systematics Of Higher Spin Gauge Fields,” *Phys. Rev.* **D21** (1980) 358; T. Curtright, “Massless Field Supermultiplets With Arbitrary Spin,” *Phys. Lett.* **B85** (1979) 219.
- [2] K. Johnson and E. C. Sudarshan, “Inconsistency Of The Local Field Theory Of Charged Spin 3/2 Particles,” *Annals Phys.* **13** (1961) 126.
- [3] G. Velo and D. Zwanziger, *Phys. Rev.* **186** (1969) 267, 1337; *ibid.* **188** (1969) 2218.
- [4] C. Aragone and S. Deser, “Consistency Problems Of Spin 2 Gravity Coupling,” *Nuovo Cim.* **B57** (1980) 33.
- [5] C. R. Hagen and L. P. Singh, “Search For Consistent Interactions Of The Rarita-Schwinger Field,” *Phys. Rev.* **D26** (1982) 393.
- [6] S. Deser, V. Pascalutsa and A. Waldron, “Massive spin 3/2 electrodynamics,” *Phys. Rev.* **D62** (2000) 105031 [hep-th/0003011].
- [7] V. Pascalutsa, “Quantization of an interacting spin-3/2 field and the Delta isobar,” *Phys. Rev.* **D58** (1998) 096002 [hep-ph/9802288].
- [8] V. Pascalutsa and R. Timmermans, “Field theory of nucleon to higher-spin baryon transitions,” *Phys. Rev.* **C60** (1999) 042201 [nucl-th/9905065].
- [9] R. D. Peccei, *Phys. Rev.* **176**, 1812 (1968); G. Höhler, in: *Landolt-Börnstein*, Vol. 9b, ed. H. Shopper (Springer, Berlin-Heidelberg, 1983); R. M. Davidson, N. C. Mukhopadhyay and R. S. Wittman, “Effective Lagrangian approach to the theory of pion photoproduction in the Delta (1232) region,” *Phys. Rev.* **D43** (1991) 71; O. Scholten, A. Yu. Korchin, V. Pascalutsa and D. Van Neck, “Pion and photon induced reactions on the nucleon in a unitary model,” *Phys. Lett.* **B384** (1996) 13; A. Yu. Korchin, O. Scholten and R. Timmermans, “Pion and photon couplings of N* resonances from scattering on the proton,” *Phys. Lett.* **B438** (1998) 1; T. Feuster and U. Mosel, “Photon and meson induced reactions on the nucleon,” *Phys. Rev.* **C59** (1999) 460; A. D. Lahiff and I. R. Afnan, “Solution of the Bethe-Salpeter equation for pion nucleon scattering,” *Phys. Rev.* **C60** (1999) 024608 [nucl-th/9903058].
- [10] F. J. Dyson, *Phys. Rev.* **73** (1948) 829; R. Haag, *Phys. Rev.* **112** (1958) 669; H. J. Borchers, *Nuovo Cimento* **25** (1960) 270; S. Kamefuchi, L. O’Raifeartaigh and A. Salam, *Nucl. Phys.* **28** (1961) 529.
- [11] S. Coleman, J. Wess and B. Zumino, “Structure Of Phenomenological Lagrangians. 1,” *Phys. Rev.* **177** (1969) 2239.
- [12] V. Pascalutsa and J. A. Tjon, “Pion nucleon interaction in a covariant hadron-exchange model,” *Phys. Rev.* **C61** (2000) 054003 [nucl-th/0003050].
- [13] H. Tang and P. J. Ellis, “Redundance of Delta isobar parameters in effective field theories,” *Phys. Lett.* **B387** (1996) 9 [hep-ph/9606432].
- [14] T. R. Hemmert, B. R. Holstein and J. Kambor, “Chiral Lagrangians and Delta(1232) interactions: Formalism,” *J. Phys. G* **24** (1998) 1831 [hep-ph/9712496].

- [15] L. M. Nath, B. Etemadi and J. D. Kimel, “Uniqueness Of The Interaction Involving Spin 3/2 Particles,” Phys. Rev. **D3** (1971) 2153.
- [16] C. R. Hagen, Phys. Rev. **D4** (1971) 2204; L. P. Singh, Phys. Rev. **D7** (1973) 1256.
- [17] C. L. Korpa, “Complete spin structure of the pion nucleon loop Delta self-energy,” Heavy Ion Phys. **5** (1997) 77 [hep-ph/9703339].
- [18] S. Kondratyuk and O. Scholten, Phys. Rev. **C62** (2000) 025203 [nucl-th/0001022].

FIGURES

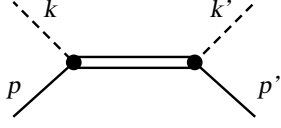


FIG. 1. The s -channel Δ -exchange in πN scattering. The dashed, solid, and double lines denote the pion, the nucleon, and the Δ , respectively.

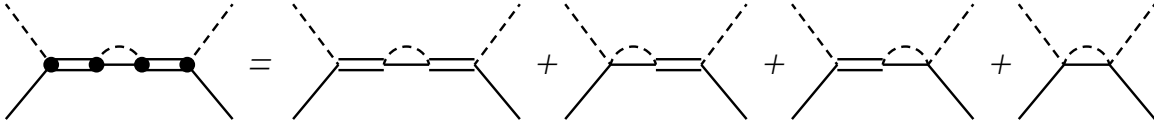


FIG. 2. The equivalence of a πN -scattering loop amplitude with the consistent $\pi N\Delta$ coupling (denoted by a dot) on the l.h.s., and in the corresponding inconsistent model on the r.h.s.; crossed graphs are omitted.